A pragmatic and unified approach to CVA, PFE and VaR

Robert Thorén, Head of Risk Solutions, Algorithmica Research
Overview

• Some definitions and questions to consider
• Going from PFE to CVA within the same calculation framework
• Some comments on computer implementation
• Questions and answers
Acronyms explained

• VaR (Value at Risk)
  – The loss amount expected to be breached a certain percentage number of days. (ex. @99% confidence level, the VaR amount will 1 day in 100 be larger than this amount, the other days less, on average, for a typical 100 day period.)

• PFE (Potential Future Exposure)
  – With a certain confidence, the estimated positive value of positions traded with a certain counterpart. (ex. given a large set of forward market states, a PFE @99% confidence means that there will a 1% probability that the current exposure will become this large.)

• CVA (Credit Value Adjustment)
  – The net present value of a all future time periods’ positive expected value, scaled with a given counterpart’s expected probability of default at each time. Further adjusted for the recovery rate of that counterpart at default.
Acronyms visualized

Value-at-risk (VaR)  Expected MtM  Expected Exposure (EE)  Potential Future Exposure (PFE)
VaR, PFE & CVA
similarities and differences

1) Given a non-parametric VaR model such as Historical Simulation, one needs historical samples to (re)price all positions.

2) Accepting historical simulation as model for forward looking risk implies;
   a) Wanting to use implied correlations (and not constant)
   b) That forward volatility can be guessed from the past
   c) An empirical distribution of returns (i.e. fat tails, skew, etc)

3) Why would a longer term forecast of Potential Future Exposure be underpinned by a different set of assumptions of the dynamics of market rates, quotes, etc? It should not. Same assumptions apply!

4) Calculation of CVA is dependent on calculation of the expected exposure. Could we take this directly from the PFE calculation?

-> Not exactly. CVA is the *market price* of counterparty risk, PFE and VaR are risk measures. Assuming you want to hedge CVA with market instruments, it needs to be calculated using risk neutral (market implied) assumptions.
Risk factors

- Typical for VaR
  - Try to use as many as possible in order to be able to correctly price positions and give many degrees of freedom of analysis.
  - Implies interest rates, inflation, basis and credit curves with n-factors (i.e. complete curve dynamics)
  - Equities represented as themselves (if possible)
  - Implied volatility surfaces as risk factors (with skew dynamics if possible)
  - Commodities as spot + convenience yields (or forwards curves)
Ok, assume we like our risk factor model ...

What kind of properties would we like to see for the representation of future market states in a multi-step forward simulation typical in a PFE real-world risk simulation?

- To have yield-curves in forward states that remain arbitrage free, and “look plausible”
- That the required relations between groups of market factors are maintained. For example, we seldom want a BBB corporate spread to become narrower than the A spread, and the A spread should not be tighter than the AA spread and so on.
- A possibility to enforce mean-reversion in rates in order to avoid them from exploding on long time-horizons
- That the generated paths exhibit the same statistical properties as the real historical series, such as autocorrelation, kurtosis and correlations
- The ability to set drift terms that are in line with long term forward views on different asset classes and individual series

One method that will give this type of flexibility, while preserving observed features in historical market data, is bootstrapping, i.e. using the actual historical return data as the random number generator.

These sample draws can then be stratified in order to re-create auto-correlation features. Yield curve nodes for each draw can be mildly “iid-perturbed”, creating a yield curve propagation that preserves the general and realistic shapes of the yield curve.
Scenario generation (details)

<table>
<thead>
<tr>
<th>Model</th>
<th>Real world</th>
<th>Risk neutral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>Drift + random</td>
<td>Hist calibrations and/or analyst judgement</td>
</tr>
<tr>
<td>Commodity</td>
<td>Drift + random (conv yield mean rev)</td>
<td>Hist calibrations and/or analyst judgement</td>
</tr>
<tr>
<td>Base yield curves</td>
<td>Mean rev in short and long end + spring constants</td>
<td>Hist calibrations and/or analyst judgement</td>
</tr>
<tr>
<td>Ir and fx basis spreads</td>
<td>Mean revertering in abs or rel spread mean. Relative restrictions</td>
<td>Hist calibrations and/or analyst judgement</td>
</tr>
<tr>
<td>Credit spreads</td>
<td>Mean revertering in abs or rel spread mean. Relative restrictions</td>
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<tr>
<td>FX</td>
<td>Drift + random, possible to mean revert</td>
<td>Hist calibrations and/or analyst judgement</td>
</tr>
<tr>
<td>Volsurfaces</td>
<td>Mean revertering ATM levels. Shape preservation.</td>
<td>Hist calibrations and/or analyst judgement</td>
</tr>
<tr>
<td>Correlations</td>
<td>Sampling implied</td>
<td>Sampling implied</td>
</tr>
</tbody>
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Real world vs. risk neutral

Example of SEK/NOK exchange rate using historical calibration (drift-less by choice)
Risk neutral drift

Example of SEK/NOK exchange rate under risk neutral drift
Long term yield curves
probably the hardest to simulate

- (Block) bootstrap as proposed by Rebonato et al.
- Using full set of factors to avoid caveats of PCA
- Use spring-constants i.e. ”arbitrageurs” to avoid intra curve anomalies
- Simultaneous calibration (mean-reverting) of short and long end of curve
- Results in well behaved curves for the very long term, but with all features of real-world dynamics such as auto-correlation, kurtosis, etc.

SEKSWAP curves
Risk neutral
1000 days forward
Calib on 500 days
Method of simulated CVA

\[ CVA \approx LossGivenDefault \times \sum_{t=0}^{T} EE(t) \times DiscountRate(t) \times DefaultProb(t) \]
How Risk Neutral Sim. is done
And does it work?

1) Specify the relevant model for all asset classes
2) Do an initial calibration in the real-world
3) Draw (block)-bootstrapped random change samples from the historical data
4) Apply random samples to the calibrated models
5) Generate many complete market paths
6) Adjust samples in each step so that the desired risk neutral dynamics are recovered
7) Price instruments either in the real world or adjusted world

Empirical evidence show that IR, Fx, and Equity derivatives can be priced using the simulated paths calibrated to market variables.

*Why is that?*

Because derivative pricing is mainly dependent on mean and variance. Risk is dependent on higher moments, tails etc.
Wrong-way risk in CVA

Explained: the fact that the exposure to a counterparty may be positively correlated with the timing/probability of a default event.

Two main ways to incorporate wwr in the model:
1) Use the stochastic CDS implied hazard rates when calculating CVA
2) Adding a wrong-way risk term individually calibrated on each counterparty to give correlation between default probability and exposure.
How to hedge CVA?

• Ensure legal netting, ideally using ”super-master netting agreements” and daily collateral margining
• Use central clearing
• Buy CDS credit protection
• Hedge underlying market exposures so that mismatches are removed
Why is computational performance an issue?

• Example portfolio of
  – 10,000 Swaps
  – 20,000 FX-forwards

• Calculate 10,000 simulation runs
  – For approx 200 risk factors

• Monthly revals and 5 year horizon
  \[30,000 \times 10,000 \times 12 \times 5 = 18,000,000,000 \text{ unique revals}\]
  \[\Rightarrow \text{Under 5 min? 60 million revals per second}\]
How to make the system go lightning fast?

RECOMMENDED STEPS
- Implement core func in c/c++
- Vectorize important PV functions
- OpenMP for parallelization
- Use Intel’s Math Kernel Library
- Multi-core processor server ex. 4x8s
- Sandy/Ivy Bridge type processor with Advanced Vector Extension (AVX)
- If needed add extra calc server(s) for additional parallelization
Summary

• Benefits of having a unified model
  – Reuse of underlying risk factor data, assumptions, connection to positions and system setup

• Calculation of PFE and CVA using either risk neutral or real world measure can be done simultaneous

• Correlation from history but not calibrated to the market; both good and bad

• Hedging market risk for VaR will have similar effect on PFE and CVA if within the same netting set.

• Stress-testing on initial market data can be done concurrent for VaR, PFE and CVA.
Thank You! Q & A?